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Stability and leptogenesis in the left-right symmetric seesaw mechanism

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Abstract: We analyze the left-right symmetric type I+II seesaw mechanism, where an eight-fold degeneracy among the mass matrices of heavy right-handed neutrinos M_R is known to exist. Using the stability property of the solutions and their ability to lead to successful baryogenesis via leptogenesis as additional criteria, we discriminate among these eight solutions and partially lift their eight-fold degeneracy. In particular, we find that viable leptogenesis is generically possible for four out of the eight solutions.

Keywords: Baryogenesis, Neutrino Physics.

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Contents

1. Introduction

In recent years, it has become an established fact that neutrinos, though relatively light, are massive. Since the first experimental evidence of neutrino oscillations until today an enormous progress has been made in determining the low-energy properties of neutrinos, such as mass squared differences and mixing. The existence of neutrino masses poses some fundamental theoretical challenges, such as understanding why the neutrino mass is so much smaller than the masses of the other fermions. An elegant and attractive solution to this problem is given by the seesaw mechanism [1 − 9], which explains the smallness of the neutrino mass through the existence of very heavy particles (usually right-handed Majorana neutrinos or Higgs triplets), the mass scale of which could be related to that of Grand Unification. In addition, the seesaw mechanism provides a natural explanation of the baryon asymmetry of the Universe through the baryogenesis via leptogenesis mechanism [10] (for recent reviews, see refs. [11 − 13]). However, the large mass scale of the seesaw particles jeopardizes the hopes of testing this mechanism in the laboratory and hence reduces its predictivity.

In the present work, we consider the seesaw mechanism in a class of left-right symmetric models in which the intermediate states with both right-handed neutrinos (type I) and heavy triplet scalars (type II) contributions to the light neutrino mass matrix m_ν are naturally present. We focus on a special case with a discrete left-right symmetry, in which type I and type II seesaw contributions contain the same triplet Yukawa coupling f. This case has much fewer parameters than the most general one and is therefore more predictive. After integrating out the heavy particles, the light neutrino mass matrix is given by

$$m_\nu = f\, v_L - \frac{v^2}{v_R}\, y\, f^{-1} y^T\,, \tag{1.1}$$

where f is the triplet Majorana-type Yukawa coupling, y is the Dirac-type Yukawa coupling of neutrinos and v, v_L, and v_R are vacuum expectation values (VEVs). The first term in eq. (1.1) is the type II contribution, while the second term is the type I contribution from the original seesaw scenario. In the case when y is a complex symmetric matrix, it was shown in ref. [14] that if the light neutrino mass matrix m_ν, the VEVs, and the Dirac-type Yukawa coupling matrix y are known, the seesaw formula (1.1) can be inverted analytically to find the triplet Yukawa coupling matrix f. Since the seesaw equation is non-linear in f, one can expect multiple solutions, and indeed an eight-fold of allowed solutions is found [14]. As the mass matrix of heavy right-handed Majorana neutrinos is given by $M_R = f v_R$, this also implies an eight-fold ambiguity for this mass matrix. For given Dirac-type Yukawa coupling matrix y and VEVs, all eight solutions for f result in exactly the same mass matrix of light neutrinos m_ν, and thus, the seesaw relation by itself does not allow one to select the true solution among the possible ones. One therefore has to invoke some additional information and/or selection criteria. The present work is an attempt in this direction.

One possibility to discriminate among the eight allowed solutions for f is to introduce a notion of naturalness. For example, for certain ranges of the VEVs and certain solutions, a very special triplet coupling matrix f might be needed, in the sense that marginally different f would lead to significantly different low-energy phenomenology. We consider such a situation unnatural; the degree of tuning that is required in the right-handed sector to obtain the observed neutrino phenomenology will be quantified and the corresponding selection criterion for f discussed in section 3.

Another possibility to discriminate among the allowed solutions is to constrain them by the phenomenology of the right-handed neutrinos. Since the right-handed sector of the theory is not directly accessible to laboratory experiments, cosmological benchmarks turn out to be the most promising tool. Namely, we will classify the solutions according to their ability to lead to successful baryogenesis via leptogenesis. This will be discussed in section 4, before we draw our conclusions in section 5.

Recently, leptogenesis in a class of models with the left-right symmetric seesaw mechanism has been considered in a similar framework in ref. [15]. We compare our results with those in ref. [15] in sections 4 and 5.

2. The model and the inversion formula

In this section, we introduce our framework and set up the notation. In the basis where the mass matrix of charged leptons is diagonal, the light neutrino mass matrix can be written as

$$m_\nu = (P_l\, U_{\rm PMNS}\, P_\nu)^*\, m_\nu^{\rm diag}\, (P_l\, U_{\rm PMNS}\, P_\nu)^\dagger\,, \tag{2.1}$$

where $m_\nu^{\rm diag} = {\rm diag}(m_1, m_2, m_3)$ is the diagonal matrix of neutrino masses, $U_{\rm PMNS}$ is the leptonic mixing matrix which depends on three mixing angles and a Dirac-type CP-violating phase, and P_l and P_ν are diagonal matrices of phase factors, which in general contain five independent complex phases.

The neutrino masses m_1, m_2, and m_3 can be expressed through the lightest neutrino mass m_0 and the two mass squared differences Δm^2_{21} and Δm^2_{31}. In our numerical calculations, we will use the current best-fit values of the parameters defining the neutrino mass matrix [16 – 18]:

$$\Delta m^2_{21} \simeq 7.9 \times 10^{-5}\ {\rm eV}^2\,, \quad \Delta m^2_{31} \simeq \pm 2.6 \times 10^{-3}\ {\rm eV}^2\,, \tag{2.2}$$

$$\theta_{12} \simeq 33.2^\circ\,, \quad \theta_{23} \simeq 45^\circ\,. \tag{2.3}$$

For the mixing angle θ_{13}, only the upper limit $\theta_{13} \lesssim 11.5^\circ$ exists. Unless explicitly stated otherwise, we will use the value $\theta_{13} = 0$ in our analysis.

We will be assuming that the Dirac-type Yukawa coupling matrix of neutrinos y coincides with that of the up-type quarks y_u. This is a natural choice in the light of quark-lepton symmetry and grand unified theories (GUTs) [19 – 21]. Actually, this relation is unlikely to hold exactly, since, in the GUT framework, it would also imply that the Yukawa couplings of the down-type quarks and charged leptons coincide, $y_d = y_l$, in contradiction with experiment. GUT models that modify this relation usually also modify the relation between the up-type and neutrino Yukawa matrices [22, 23]. However, most of the qualitative results in the present work depend only on the fact that the eigenvalues of y are hierarchical. Whenever a result relies on the assumption $y = y_u$, we will comment explicitly on this issue. Following ref. [14], we will also assume y to be symmetric. In this case, the two VEVs (v_L and v_R), the sign of Δm^2_{31}, and the mass scale of the light neutrinos are the only free parameters (ignoring for the moment the CP-violating phases, which will be discussed in section 4).

Our choice of the Dirac-type Yukawa coupling matrix implies that it can be written as

$$y = P_d\, U^T_{\rm CKM}\, P_u\, y_u^{\rm diag}\, P_u\, U_{\rm CKM}\, P_d\,, \tag{2.4}$$

where the eigenvalues of $y_u^{\rm diag}$ are

$$y_u^{\rm diag} = {\rm diag}(4.2 \times 10^{-6},\, 1.75 \times 10^{-3},\, 0.7)\,, \tag{2.5}$$

and we use the standard parameterization of the CKM matrix $U_{\rm CKM}$ [24] with

$$\theta^q_{12} \simeq 13.0^\circ, \quad \theta^q_{13} \simeq 0.2^\circ, \quad \theta^q_{23} \simeq 2.2^\circ, \quad \delta^q \simeq 1.05\,. \tag{2.6}$$

The values in eqs. (2.5) and (2.6) are evaluated at the GUT scale, following ref. [15]. The matrices P_u and P_d in eq. (2.4) are diagonal matrices of phase factors. The phases in the four matrices P_l, P_ν, P_u, and P_d are partially redundant. For example, by a redefinition of the fields, the three phases of P_l can be moved into P_d, so that we are left with the two usual Majorana phases and the Dirac phase in the low-energy sector, while five additional Majorana phases and one Dirac phase reside in y and can only affect high-energy processes

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such as leptogenesis. Even though these phases can marginally influence the stability of the seesaw solutions, we set the high-energy phases to zero in the first part of our work and consider them only in the part where leptogenesis is discussed.

In order to invert the seesaw formula, it is useful to introduce the following dimensionful quantities:

$$g = v_L\, f\,, \qquad \mu = \frac{v_R}{v_L\, v^2}\,, \tag{2.7}$$

with the VEV $v \simeq 174\,\mathrm{GeV}$, so that eq. (1.1) turns into

$$m_\nu = g - \frac{1}{\mu}\, y\, g^{-1} y^T. \tag{2.8}$$

This convention has the advantage that the matrix g will only depend on μ and not on the two VEVs, v_L and v_R, separately. It will turn out that the baryon asymmetry produced via leptogenesis depends only on this combination of VEVs, so that, besides the CP-violating phases, we are left with two parameters only, the quantity μ and the lightest neutrino mass m_0. The hierarchy of the light neutrino masses can be considered as an additional discrete parameter.

In the following, we give a short description of the seesaw inversion formula from refs. [14, 25] in the case of three lepton generations and when y is a complex symmetric matrix. In the basis where y is diagonal, the seesaw equation for g reduces to the following system of six coupled non-linear equations for its matrix elements g_{ij}:

$$\mu G[g_{ij} - (m_\nu)_{ij}] = y_i y_j G_{ij}\,. \tag{2.9}$$

Here we use the notation

$$G \equiv \det g, \qquad G_{ij} = \frac{1}{2} \sum_{k,l,m,n=1}^{3} \epsilon_{ikl}\epsilon_{jmn} g_{km} g_{ln}\,. \tag{2.10}$$

It was found in ref. [14] that in the case when y is symmetric, for every solution g there exists another solution $\tilde{g}$ which is related to g by the duality transformation $\tilde{g} = m_\nu - g$. For $\tilde{g}$, eq. (2.9) reads

$$\mu\tilde{G}[\tilde{g}_{ij} - (m_\nu)_{ij}] = -\mu\tilde{G} g_{ij} = y_i y_j \tilde{G}_{ij} \tag{2.11}$$

with $\tilde{G} \equiv \det \tilde{g}$. The system of equations in eq. (2.9) can now be solved by making use of the following procedure. First, we introduce the rescaled matrices $g' = g/\lambda^{1/3}$, $m'_\nu = m_\nu/\lambda^{1/3}$, and $y' = y/\lambda^{1/3}$, where λ is to be determined from the equation $G'(\lambda) \equiv \det g'(\lambda) = 1$. Then, using the equation for the dual quantities $\tilde{g}'$, one can linearize the system of equations for g'_{ij}. Next, this system can be solved and one obtains the following solution for g:

$$g_{ij} = \frac{\lambda^2[(\lambda^2 - Y^2)^2 - Y^2\lambda \det m_\nu + Y^4 S](m_\nu)_{ij} + \lambda(\lambda^4 - Y^4)A_{ij} - Y^2\lambda^2(\lambda^2 + Y^2)S_{ij}}{(\lambda^2 - Y^2)^3 - Y^2\lambda^2(\lambda^2 - Y^2)S - 2Y^2\lambda^3 \det m_\nu}\,, \tag{2.12}$$

where

$$Y^2 \equiv \frac{(y_1 y_2 y_3)^2}{\mu^3}, \quad S \equiv \mu \sum_{k,l=1}^{3} \left[\frac{(m_\nu)^2_{kl}}{y_k y_l} \right], \quad A_{ij} \equiv \frac{y_i y_j M_{ij}}{\mu}, \tag{2.13}$$

$$S_{ij} \equiv \mu \sum_{k,l=1}^{3} \left[(m_\nu)_{ik} (m_\nu)_{jl} \frac{(m_\nu)_{kl}}{y_k y_l} \right] \tag{2.14}$$

with $M_{ij} = \frac{1}{2}\epsilon_{ikl}\epsilon_{jmn}(m_\nu)_{km}(m_\nu)_{ln}$. In terms of the original (non-rescaled) quantities, one has $G(\lambda) \equiv \det g(\lambda) = \lambda$, which yields an eighth order equation for λ. Using the duality property, one can reduce it to a pair of fourth order equations. Substituting the solutions for λ into eq. (2.12) gives eight solutions for g_{ij}. In general, for n lepton generations the number of solutions is 2^n [14].

The matrix structure of the solutions of the seesaw equation was studied in some detail in ref. [25]. In the present work, we will rather focus on the eigenvalues of the matrices g, the corresponding mixing parameters, stability properties of the solutions, and the implications for leptogenesis.

3. Stability analysis

Since the neutrino Dirac-type Yukawa coupling matrix in our framework is given by the up-type quark mass matrix, the inversion formula of the previous section can be used to determine the eight allowed structures of the triplet coupling matrix $f = g/v_L$ for given low-energy neutrino mass matrix m_ν and the parameters v_L, v_R, and m_0. Our stability analysis is based on the assumption that the Dirac-type coupling matrix y and the Majorana-type coupling matrix f are *a priori* independent (for a discussion of the situations when this is not the case, see section 5 of ref. [25]). We pose the question of whether the resulting low-energy phenomenology is stable under small changes in f. Since the inversion formula in general yields eight valid solutions, the mass matrix m_ν and the corresponding Majorana coupling matrix f are in a 1-to-8 correspondence. It is still a reasonable question to ask if for the measured m_ν some of the predicted f have to be very special, so that a fine-tuning is required and a small modification of their elements may lead to a large change in $(m_\nu)_{ij}$.

The measure we use to quantify the stability property of the solutions is the following:

$$Q = \left| \frac{\det f}{\det m_\nu} \right|^{1/3} \sqrt{\sum_{k,l=1}^{2N} \left(\frac{\partial m_l}{\partial f_k} \right)^2}. \tag{3.1}$$

The real coefficients f_k and m_l determine the matrices f and m_ν according to

$$f = \sum_k (f_k + i f_{k+N}) T_k, \tag{3.2}$$

$$m_\nu = \sum_k (m_k + i m_{k+N}) T_k, \tag{3.3}$$

where $T_k,\ k \in [1,N]$ with $N = n(n+1)/2$, form a basis of complex symmetric $n \times n$ matrices. For this basis, we choose the normalization

$$\mathrm{tr}\,(T_l^\dagger\, T_k) = \delta_{lk}\,. \tag{3.4}$$

The resulting stability measure Q does not depend on the chosen basis. This can be easily seen in the following way. Consider another basis T'_k satisfying eq. (3.4). The two bases are then connected via a unitary transformation $T'_k = \sum_l U_{kl}\, T_l$. The coefficients in the old and new bases are determined as

$$f_k = \mathrm{Re}\left[\mathrm{tr}\,(T_k^\dagger\, f)\right], \quad f_{k+N} = \mathrm{Im}\left[\mathrm{tr}\,(T_k^\dagger\, f)\right], \tag{3.5}$$

$$f'_k = \mathrm{Re}\left[\mathrm{tr}\,(T_k'^\dagger\, f)\right], \quad f'_{k+N} = \mathrm{Im}\left[\mathrm{tr}\,(T_k'^\dagger\, f)\right], \tag{3.6}$$

and hence, are related by an orthogonal transformation

$$f'_a = \sum_b O_{ab}\, f_b, \quad a,b \in [1,2N], \quad O = \begin{pmatrix} \mathrm{Re}\, U & \mathrm{Im}\, U \\ -\mathrm{Im}\, U & \mathrm{Re}\, U \end{pmatrix}, \tag{3.7}$$

which leaves the measure in eq. (3.1) invariant.[1]

Many interesting properties of the seesaw inversion formula appear already in the one-flavor case. The solutions g are then given by

$$g = \frac{m_\nu}{2} \pm \sqrt{\frac{m_\nu^2}{4} + \frac{y^2}{\mu}} \tag{3.8}$$

and our stability measure simplifies to

$$Q = f\,\frac{d}{df}\log|m_\nu| = g\,\frac{d}{dg}\log|m_\nu| = \sqrt{1 + \frac{4\,y^2}{\mu m_\nu^2}}\,. \tag{3.9}$$

In the following, we will discuss the qualitative behavior of the solutions f in various regions of the parameter space and its implications for the stability of these solutions.

3.1 Large μ regime

In the regime of large μ,

$$\mu \gg \frac{4y^2}{m_\nu^2}\,, \tag{3.10}$$

the two solutions in the one-flavor case are given by

$$g \to -\frac{y^2}{\mu m_\nu} \quad \text{and} \quad g \to m_\nu\,. \tag{3.11}$$

In this regime, the solutions are purely type I or type II dominated. In the three-flavor case, the eight solutions follow from the eight corresponding choices for the eigenvalues and

[1] Note that the stability issue was also discussed in ref. [15] where a different stability criterion, constraining only the element f_{33}, was introduced.

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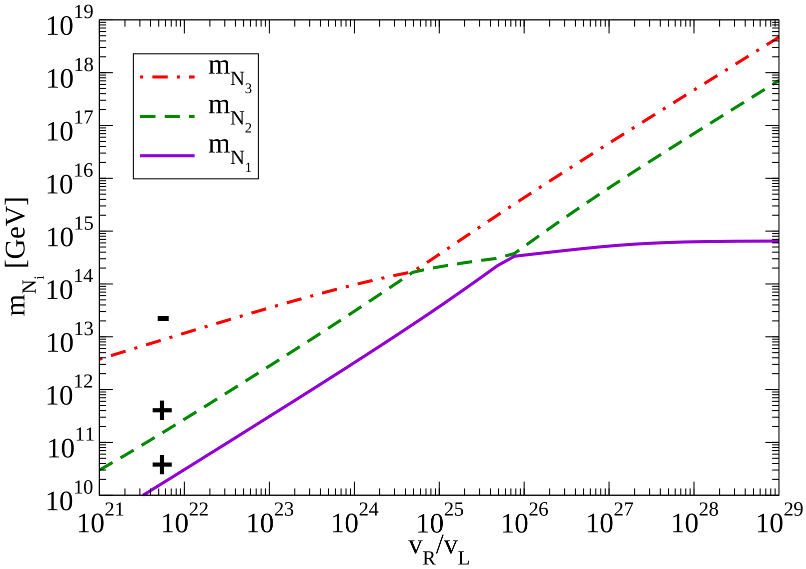

Figure 1: An example of our labeling convention for the solution '$-++$'.

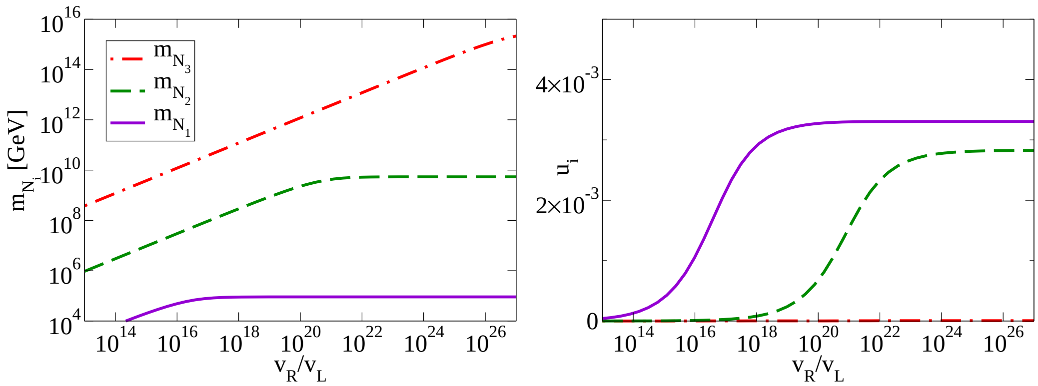

Figure 2: The right-handed neutrino masses m_{N_i} and mixing parameters u_i as functions of v_R/v_L for the solution '$---$'. Normal mass hierarchy, $m_0 = 0.001\,\mathrm{eV}$.

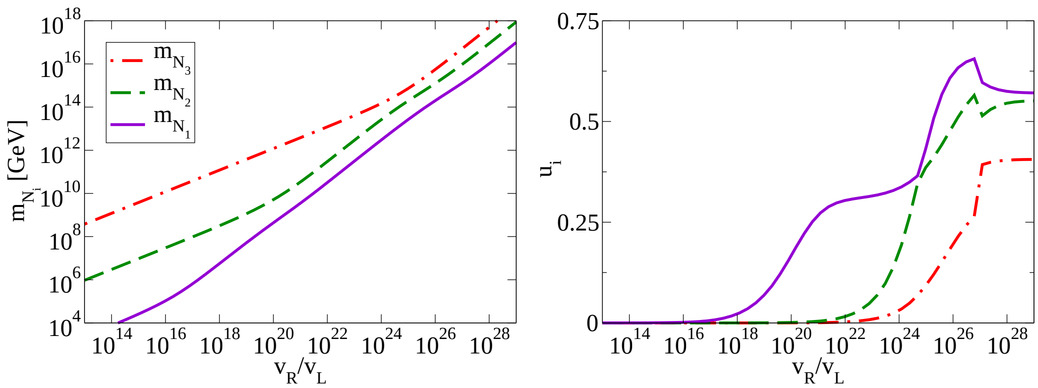

Figure 3: Same as in figure 2, but for the solution '$+++$'.

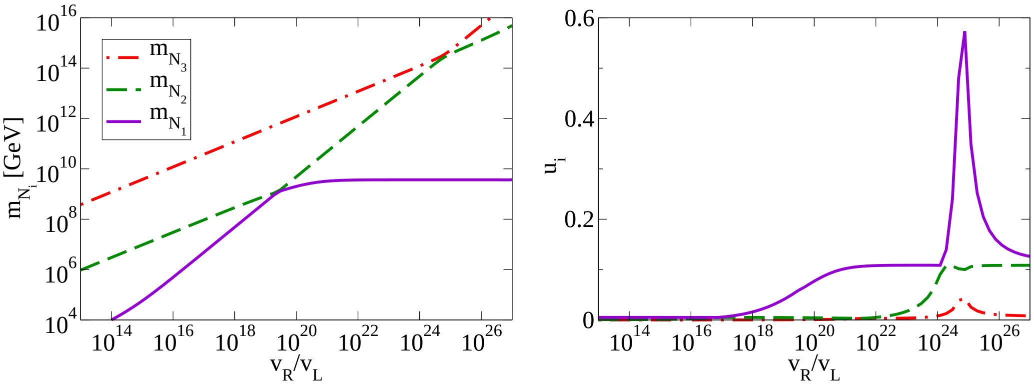

Figure 4: The right-handed neutrino masses m_{N_i} and mixing parameters u_i as functions of v_R/v_L for the solution '$+-+$'. Inverted mass hierarchy, $m_0 = 0.001\,\mathrm{eV}$.

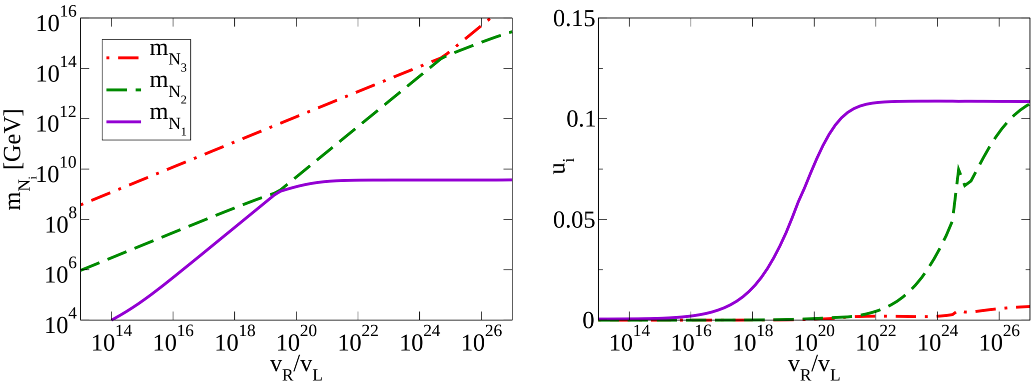

Figure 5: Same as in figure 4, but for the solution '$--+$'.

we will label these solutions according to their limiting behavior at large μ as '$-$' or '$+$' in the case of type I or type II dominance (starting with the largest eigenvalue in the small μ regime). This notation agrees with the one used in ref. [15]. Our convention is illustrated in figure 1 using the solution '$-++$' as an example.

From eq. (3.9) one can observe that in the large μ regime of the one-flavor case, both solutions for g are characterized by the stability measure $Q \simeq 1$, which is a very stable situation. Note that for the three-flavor case, no fine-tuning corresponds to $Q \simeq 10$. However, for several flavors and hierarchical y, there is in general an instability related to mixing that will be discussed in the next subsection.

3.2 Hierarchy induced large mixing

For simplicity, we start with a discussion of the two-flavor case in the pure type I seesaw framework. By hierarchy induced large mixing we mean the following: Suppose that y has a hierarchical structure

$$y \sim \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.12}$$

while, in contrast to this, the low-energy neutrino mass has a rather mild or even no hierarchy. Then, the corresponding matrix g is necessarily characterized by the hierarchy that is the squared hierarchy of y. Indeed, introducing a unitary matrix $U(\theta)$ that diagonalizes g, one finds

$$g = -\frac{1}{\mu} y\, m_\nu^{-1}\, y = U^\dagger(\theta)\, \hat{g}\, U^*(\theta) \tag{3.13}$$

with the diagonal matrix

$$\hat{g} \sim \begin{pmatrix} \epsilon^2 & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.14}$$

and, in addition, mixing has to be small, i.e. $\theta \sim \epsilon$. This was already observed in refs. [26–28] and suggested as a possible mechanism for generating large mixing angles in the light neutrino mass matrix out of small mixing angles in the right-handed and Dirac sectors. However, in our context, this is not a desirable situation, since it would require a fine-tuning between the Dirac and Majorana Yukawa couplings, i.e. between the sectors that we have assumed to be unrelated. In terms of stability, this would lead to large values of Q. In addition, the large hierarchy among the elements of the Dirac-type Yukawa coupling matrix y would induce a huge hierarchy among the elements of g, leading in general to an extremely small mixing in the right-handed neutrino sector, which may preclude successful leptogenesis.

The above consideration was based on the type I seesaw formula, and hence, is not fully applicable to our framework. Still, it applies to the solutions dominated by type I seesaw. Figure 2 shows the one out of the eight solutions that is fully dominated by the type I term in the large μ regime and is labeled as '$---$'. As a measure of mixing, we consider the parameters u_i which are related to the off-diagonal elements of the unitary matrix U diagonalizing g as follows:[2]

$$u_1^2 = \frac{1}{2}(|U_{12}|^2 + |U_{21}|^2)\,, \quad u_2^2 = \frac{1}{2}(|U_{13}|^2 + |U_{31}|^2)\,, \quad u_3^2 = \frac{1}{2}(|U_{23}|^2 + |U_{32}|^2)\,. \tag{3.15}$$

These parameters, along with the masses of right-handed neutrinos, are plotted for several solutions in figures 2–5.

For the solution '$---$', mixing is small in the large μ regime, as can be seen from figure 2. For the other seven solutions, this does not hold in general, as can be seen e.g. in figure 3. However, even in the general case, one feature seems to be universal: If the matrix elements of g exhibit a strong hierarchy, then the mixing in the right-handed sector is suppressed, which leads to the necessity of fine-tuning between the Dirac and Majorana sectors and related instabilities. This also explains why the two solutions '$++-$' and '$+--$' are very unstable with almost equal stability measure Q. The strong hierarchy between the largest and smallest right-handed masses leads to large instabilities, while the behavior of the third mass is rather irrelevant.

[2]Recall that we work in the basis where the matrix y is diagonal.

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3.3 Small μ regime

When μ is small in the sense that

$$\mu \ll \frac{4y^2}{m_\nu^2}\,, \tag{3.16}$$

in the one-flavor case, one finds the following limiting behavior for g:

$$g \to \pm\frac{y}{\sqrt{\mu}} + \frac{m_\nu}{2} + \mathcal{O}(\sqrt{\mu}), \quad \mu \to 0. \tag{3.17}$$

For the stability measure, eq. (3.9) gives

$$Q = \left|\frac{g}{m_\nu}\frac{dm_\nu}{dg}\right| \to \frac{2y}{\sqrt{\mu}\,m_\nu} \to \infty \tag{3.18}$$

in this limit, and therefore a very unstable situation. This had to be expected, since there is an almost exact cancellation between the type I and type II contributions to m_ν in the seesaw formula in this regime. In the multi-flavor case, there is an additional instability in the small μ limit which stems from the fact that mixing in g is suppressed by the hierarchy in y. This can be illustrated by the two-flavor case, in which the four solutions are of the form

$$g = \frac{1}{\sqrt{\mu}} y^{1/2} P y^{1/2} \tag{3.19}$$

with P of the form

$$P \;\propto\; \pm\,\mathbb{1} + \mathcal{O}(\sqrt{\mu}) \quad \text{or} \quad P \;\propto\; \pm\begin{pmatrix}\cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha\end{pmatrix} + \mathcal{O}(\sqrt{\mu}). \tag{3.20}$$

For the first pair of solutions, mixing vanishes in the limit $\mu \to 0$, while for the second pair, mixing in g is suppressed by the hierarchy in y. A similar argument applies to the three-flavor case and can be observed in our numerical results. For example, this behavior can be seen in figures 2 and 3 which display two out of the eight solutions for the normal mass hierarchy and $m_0 = 0.001\,\text{eV}$.

3.4 Numerical results

Figures 6 and 7 show the stability measure Q for small and large m_0 and normal/inverted mass hierarchy. For small m_0, the transition from the large μ to the small μ regime appears for larger values of μ, in accordance with eqs. (3.10) and (3.16). In all four scenarios, the solutions are unstable in the regime of small μ, which is due to the cancellation between type I and type II contributions to the mass matrix of light neutrinos. In addition, the solutions where the smallest eigenvalue is dominated by type I seesaw in the large μ regime ('$\pm\pm-$'), are unstable for large μ as well, since the lightest right-handed mass stays below 10^6 GeV in this limit and this generally leads to a large spread in the eigenvalues and to instabilities, as explained in the previous sections. Examples of the eigenvalues in these cases are given in figure 2. Analogously, the stability measure of the solutions '$\pm-+$' increases for $v_R/v_L \gtrsim 10^{20}$, since the smallest right-handed neutrino mass approaches its asymptotic value of about 10^9 GeV, as can be seen in figures 4 and 5. A similar effect

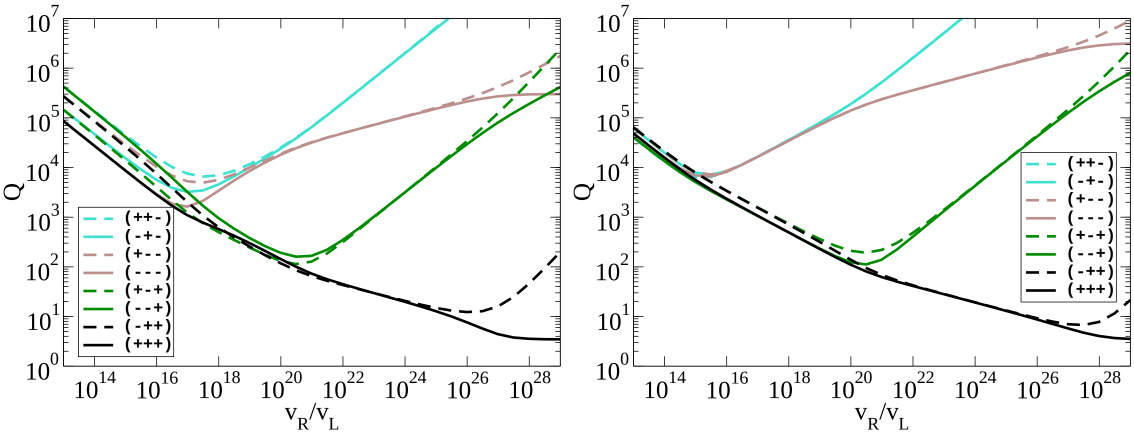

Figure 6: The stability measure Q as a function of v_R/v_L for $m_0 = 0.001$ eV. The left (right) panel corresponds to the normal (inverted) neutrino mass hierarchy.

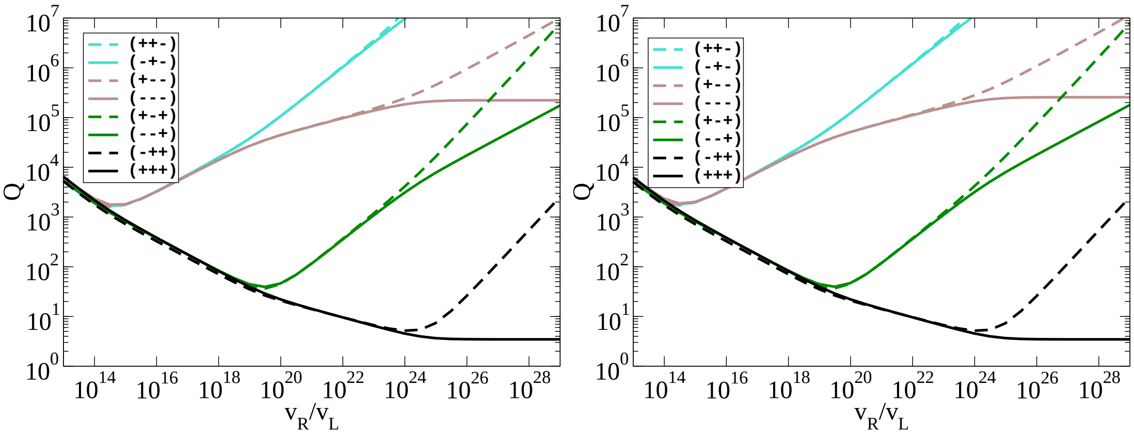

Figure 7: Same as in figure 6, but for $m_0 = 0.1\,\text{eV}$.

appears for the solution '$-++$' at values $v_R/v_L \gtrsim 10^{24}$. The purely type II dominated solution ('$+++$') is the most stable one in almost all the cases. If one allows for a tuning at a percent level, $Q \lesssim 10^3$, then the stability analysis favors the two solutions '$\pm++$' with $v_R/v_L \gtrsim 10^{18}$ and the two solutions '$\pm-+$' with $v_R/v_L \simeq 10^{20}$.

It should be noted that the qualitative behavior of the stability measure Q depends mostly on the eigenvalues of the Yukawa coupling matrix y and the neutrino mass scale m_0. On the other hand, the mixing in y and additional CP-violating Majorana phases influence the results only marginally.

4. Leptogenesis

In this section, we present our analysis of leptogenesis and its implications for the discrimination among the eight allowed solutions for g. Our analysis is based on the results of refs. [29, 30].

Assuming that the lightest of the right-handed neutrinos is separated from the other two as well as from the Higgs triplets by a large mass gap, the baryon asymmetry arising from leptogenesis can be written as

$$\eta_B \equiv \frac{n_B}{n_\gamma} = \eta\,\epsilon_{N_1}. \tag{4.1}$$

The observed value of the baryon asymmetry is $\eta_B = (6.1 \pm 0.2) \times 10^{-10}$ [31]. In eq. (4.1), η is the so-called efficiency factor that takes into account the initial density of right-handed neutrinos, the deviation from equilibrium in their decay and washout effects, while ϵ_{N_1} denotes the lepton asymmetry produced in the decay of the lightest right-handed neutrino. For the decay of the ith right-handed neutrino, it is defined as

$$\epsilon_{N_i} = \frac{\Gamma(N_i \to l\,H) - \Gamma(N_i \to \bar{l}\,H^*)}{\Gamma(N_i \to l\,H) + \Gamma(N_i \to \bar{l}\,H^*)}\,. \tag{4.2}$$

If the two lightest right-handed neutrinos have similar masses, eq. (4.1) is generalized to

$$\eta_B = \eta_1\,\epsilon_{N_1} + \eta_2\,\epsilon_{N_2}\,. \tag{4.3}$$

The coefficients η_i mostly depend on the effective neutrino masses, defined as

$$\tilde{m}_i = \frac{v^2\,(\hat{y}^\dagger\hat{y})_{ii}}{2m_{N_i}}\,. \tag{4.4}$$

Here and below, the hat indicates that the matrices are evaluated in the basis where the triplet Yukawa coupling matrix g is diagonal with real and positive eigenvalues. In the case of quasi-degenerate right-handed neutrinos, $m_{N_1} \simeq m_{N_2}$, and nearly coinciding effective masses $\tilde{m}_1$ and $\tilde{m}_2$, an order-of-magnitude estimate of the washout coefficients gives [32]

$$\eta_i \simeq \frac{1}{200}\left(\frac{10^{-3}\,\text{eV}}{\tilde{m}_i}\right). \tag{4.5}$$

However, deviations from the condition $\tilde{m}_1 \simeq \tilde{m}_2$ can lead to large corrections to this estimate. In particular, a large effective mass $\tilde{m}_2$ reduces the coefficient η_1 close to the mass degeneracy point, as is shown in figure 8. The results in ref. [32] have been obtained for rather light and quasi-degenerate right-handed neutrinos, $m_{N_1} \simeq m_{N_2} \sim 1\,\text{TeV}$. For hierarchical right-handed neutrino masses $m_{N_1} \ll m_{N_2}$ and the mass scale under consideration in the present case, $m_{N_1} \sim 10^8\,\text{GeV}$, one finds

$$\eta_1 = 1.45 \times 10^{-2}\left(\frac{10^{-3}\,\text{eV}}{\tilde{m}_1}\right), \qquad \eta_2 \simeq 0\,, \tag{4.6}$$

and we will employ these values in the following. This result and figure 8 have been obtained by solving the Boltzmann equations as suggested in ref. [32] and assuming thermal initial abundance of right-handed neutrinos.

With the washout factors η_i at hand, the determination of the baryon asymmetry requires only the knowledge of the CP-violating decay asymmetries of the right-handed

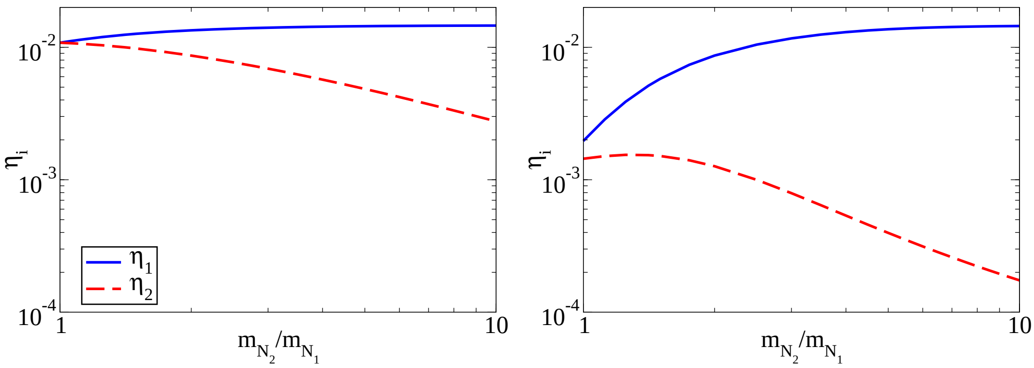

Figure 8: The contributions η_1 and η_2 to the baryon-to-photon ratio from the decays of the two lightest right-handed neutrinos versus the ratio of their masses m_{N_2}/m_{N_1}. Left panel: $\tilde{m}_1 = \tilde{m}_2 = 10^{-3}\,\mathrm{eV}$, right panel: $\tilde{m}_1 = 10^{-3}\,\mathrm{eV}$, $\tilde{m}_2 = 10^{-2}\,\mathrm{eV}$.

neutrinos ϵ_{N_i}. In the case when the low-energy limit of the theory is the Standard Model, ϵ_{N_1} is given by [30]

$$\epsilon_{N_1} = \epsilon^I_{N_1} + \epsilon^{II}_{N_1}\,, \tag{4.7}$$

$$\epsilon^I_{N_1} = \frac{1}{8\pi}\sum_{j\neq 1}\frac{\mathrm{Im}[(\hat{y}^\dagger\hat{y})^2_{1j}]}{(\hat{y}^\dagger\hat{y})_{11}}\sqrt{x_j}\left(\frac{2-x_j}{1-x_j}-(1+x_j)\ln\frac{x_j+1}{x_j}\right)\,, \tag{4.8}$$

$$\epsilon^{II}_{N_1} = \frac{3}{8\pi}\hat{g}_{11}\mu\frac{\mathrm{Im}[(\hat{y}^\dagger\,\hat{g}\hat{y}^*)_{11}]}{(\hat{y}^\dagger\hat{y})_{11}}\,z\left(1-z\,\ln\frac{z+1}{z}\right), \tag{4.9}$$

and analogous formulas hold for ϵ_{N_2}. Here $z = m^2_\Delta/m^2_{N_1}$, and x_j is defined as the ratio of the squared right-handed neutrino masses:

$$x_j = \frac{\hat{g}^2_{jj}}{\hat{g}^2_{11}}. \tag{4.10}$$

In the following, we discuss only the limit of a very heavy $SU(2)_L$ Higgs triplet, $z \to \infty$, so that

$$\epsilon^{II}_{N_1} \to \frac{3}{16\pi}\hat{g}_{11}\mu\frac{\mathrm{Im}[(\hat{y}^\dagger\,\hat{g}\hat{y}^*)_{11}]}{(\hat{y}^\dagger\hat{y})_{11}}. \tag{4.11}$$

In the limit of a strong hierarchy in the right-handed sector, $x_j \gg 1$, the first contribution in eq. (4.7) can be rewritten as

$$\epsilon^I_{N_1} \to -\frac{1}{8\pi}\sum_{j\neq 1}\frac{\mathrm{Im}[(\hat{y}^\dagger\hat{y})^2_{1j}]}{(\hat{y}^\dagger\hat{y})_{11}}\frac{3}{2\sqrt{x_j}} = -\frac{3}{16\pi}\hat{g}_{11}\frac{\mathrm{Im}[(\hat{y}^\dagger\,\hat{y}\hat{g}^{-1}\hat{y}^T\hat{y}^*)_{11}]}{(\hat{y}^\dagger\hat{y})_{11}}, \tag{4.12}$$

so that

$$\epsilon_{N_1} = \epsilon^I_{N_1} + \epsilon^{II}_{N_1} \to \frac{3}{16\pi}\hat{g}_{11}\mu\frac{\mathrm{Im}[(\hat{y}^\dagger\,\hat{m}_\nu\hat{y}^*)_{11}]}{(\hat{y}^\dagger\hat{y})_{11}}\,. \tag{4.13}$$

However, even in this limit, this approximation can lead to large deviations from the exact result of eqs. (4.7)–(4.9). Consider e.g. the regime of small μ, where type I and type II seesaw contributions almost cancel each other in the expression for the light neutrino

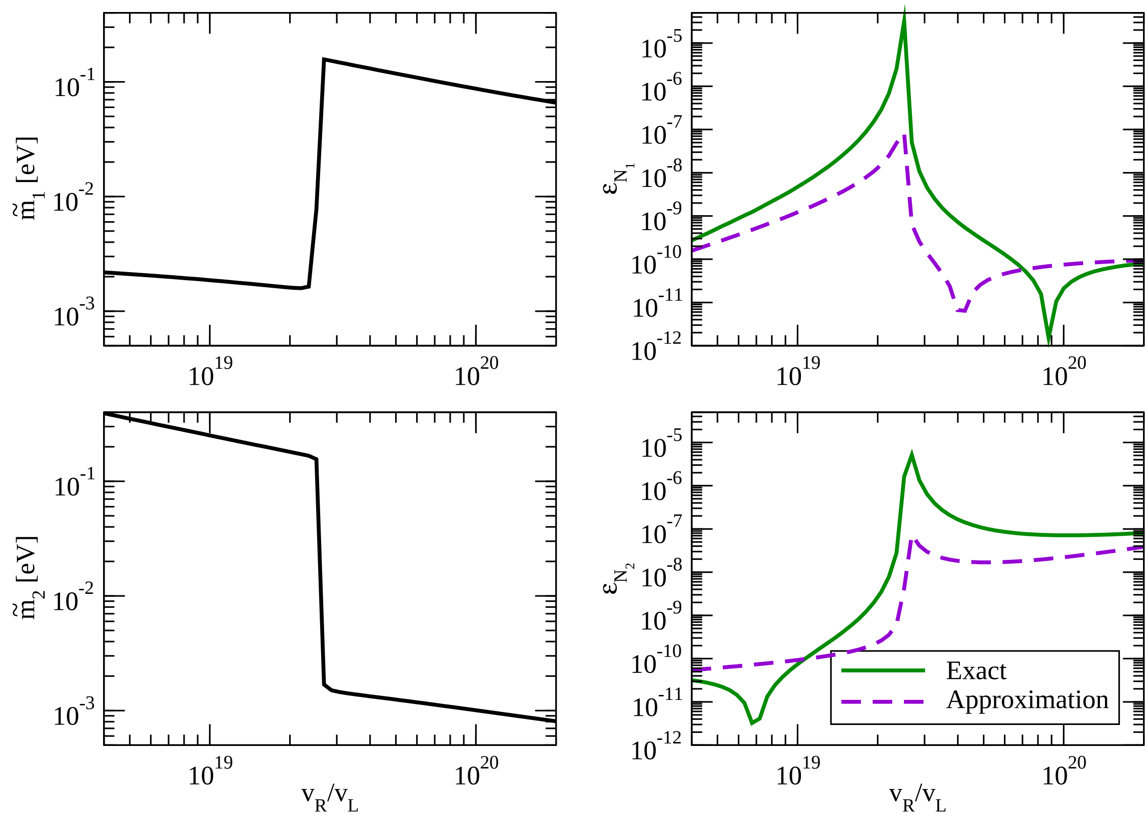

Figure 9: The upper (lower) panels show the effective neutrino mass $\tilde{m}_1$/eV ($\tilde{m}_2$/eV) and the asymmetry ϵ_{N_1} (ϵ_{N_2}) as functions of v_R/v_L for the solution '$+-+$'. The dashed curves in the right panels correspond to the approximation in eq. (4.13), while the solid curves represent the exact result. The step-like behavior of $\tilde{m}_1$ and $\tilde{m}_2$ is due to the level crossing. Inverted mass hierarchy, $m_0 = 0.001\,\text{eV}$.

mass matrix. In this case, even a small correction to the coefficient of the asymmetry $\epsilon^I_{N_1}$ leads to an incomplete cancellation and to large errors in the approximation of eq. (4.13). This effect is also partially present at intermediate values of μ. In addition, close to the mass degeneracy ($x_j \simeq 1$), a resonant feature is expected in $\epsilon^I_{N_1}$, which can lead to successful leptogenesis even at a TeV scale [32]. This is demonstrated in figure 9, where the asymmetries ϵ_{N_1} and ϵ_{N_2} produced in the decays of the two lightest right-handed neutrinos and the corresponding effective mass parameters $\tilde{m}_1$ and $\tilde{m}_2$ are plotted. The results show sizable deviations from the approximation (4.13), even outside the resonant enhancement region. The corresponding baryon-to-photon ratio is shown in figure 10. In addition, this figure shows the baryon-to-photon ratio in the case of non-vanishing θ_{13} and the Dirac-type leptonic CP-violating phase $\delta_{\text{CP}} = 30°$. The resonant behavior is less distinct for larger values of θ_{13}, which can be traced back to the fact that the two lightest right-handed neutrinos never become exactly degenerate in mass in this case. On the other hand, the Dirac-type phase constitutes an additional source of CP violation in the case of non-vanishing θ_{13}, leading to an enhancement of ϵ_{N_1} below the mass degeneracy point for

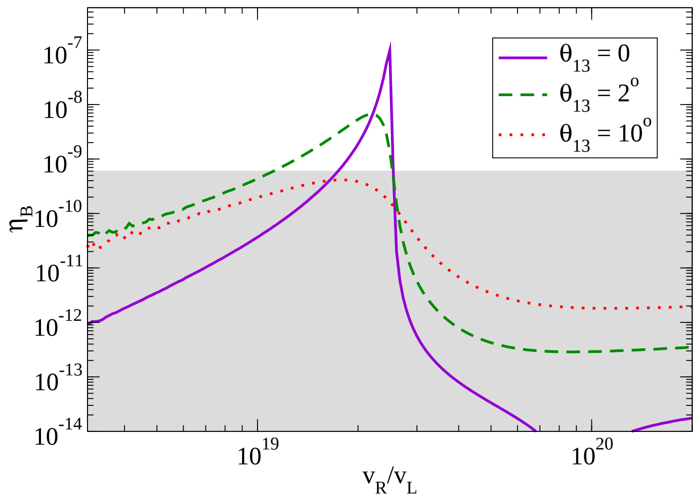

Figure 10: The baryon-to-photon ratio η_B from the decay of the lightest right-handed neutrino for the solution '$+-+$'. The same parameters as in figure 9, except that the dashed and dotted curves correspond to nonzero θ_{13} and $\delta_{\rm CP} = 30°$. The shaded area corresponds to values of η_B below the observed value.

smaller values of θ_{13}, and thus, widening the v_R/v_L region where successful leptogenesis is possible (see the dashed curve in figure 10).

Thus, we find that viable leptogenesis is possible in this scenario if the ratio of the VEVs is close to $v_R/v_L \simeq (1 \div 2) \times 10^{19}$. Note that leptogenesis in the case of the left-right symmetric seesaw mechanism was previously considered in a similar framework in ref. [15]. For the specific choice of the parameters made there, the washout processes were found to be too strong to allow successful leptogenesis. However, for our choice of the parameters with the inverted mass hierarchy in the light neutrino sector, the drop in the effective mass $\tilde{m}_1$ below the level crossing point of the two lightest right-handed neutrinos resolves this issue. We notice that the use of the exact formulas (4.7-4.9) rather than the approximation (4.13) is essential in this region.

It should be also noted that a similar effect of incomplete cancellation can appear if the mass of the Higgs triplet is of the same order as the mass of the lightest right-handed neutrino. In this case, the asymmetry $\epsilon_{N_1}^{II}$ is modified and the cancellation between type I and type II contributions is incomplete as well, which in the small and intermediate μ regimes can enhance the produced lepton asymmetry by several orders of magnitude compared to the approximation in eq. (4.13).

With the parameters of figure 10, the lightest right-handed neutrino has a mass of order $m_{N_1} \simeq 5 \times 10^9\,$GeV, as can be seen in figure 4. Since thermal leptogenesis requires a reheating temperature $T \gtrsim M_{N_1}$, this can potentially lead to a tension with bounds coming from gravitino cosmology in supersymmetric theories, namely $T \lesssim (10^7 \div 10^{10})\,$GeV [33]. Thus, this possibility imposes constraints which are similar to those in the usual pure type I seesaw scenario.

Another difference from the standard leptogenesis scenario is the appearance of the phases contained in P_ν, P_l, P_u, and P_d in the neutrino mass matrix m_ν and in the Dirac

Yukawa coupling matrix y, which up to now have been set to zero in our discussion. Due to these phases and an interplay between type I and type II contributions to the neutrino mass matrix, leptogenesis is possible, in principle, even in the case of one leptonic flavor, as will be demonstrated below. This case is quite similar to the framework with three left-handed neutrinos and one right-handed neutrino discussed in ref. [34] (see also ref. [25]). In the following, we will present some analytic results for the left-right symmetric one- and two-flavor cases, before presenting numerical results for the three-flavor case.

In the one-flavor case, the light neutrino mass is given by

$$m_\nu = g - \frac{y^2}{\mu g} \tag{4.14}$$

and the lepton asymmetry produced in the decay of the heavy right-handed neutrino is

$$\epsilon = \frac{3}{32\pi} \frac{\mathrm{Im}[\hat{y}^{*2}\hat{m}_\nu]}{\tilde{m}} \,. \tag{4.15}$$

Once again, the hat indicates that y and m_ν are in the basis where g is real and positive. It turns out that the most interesting regime is given by large values of μ and a relative phase of $\pi/4$ between m_ν and y. In this case, only the solution dominated by the type II term is relevant, since the type I contribution to $\hat{y}^{*2}\hat{m}_\nu$ is real and cannot generate any CP asymmetry. Thus, we obtain

$$g \simeq m_\nu, \quad m_N = m_\nu \mu v^2 \,, \tag{4.16}$$

and

$$\tilde{m} = \frac{|y|^2\, v^2}{2m_N} = \frac{|y|^2}{2\, m_\nu \mu} \,, \tag{4.17}$$

$$\epsilon = \frac{3}{16\pi} m_\nu^2 \mu = \frac{3}{16\pi} \frac{m_\nu m_N}{v^2} \,, \tag{4.18}$$

$$\eta_B = 1.7 \times 10^{-6}\,\mathrm{eV}\, \frac{m_\nu^3 \mu^2}{|y|^2} = 1.7 \times 10^{-6}\,\mathrm{eV}\, \frac{m_\nu m_N^2}{|y|^2\, v^4} \,. \tag{4.19}$$

Thus, it is possible to reproduce the observed baryon asymmetry e.g. with the values

$$|y| = 10^{-4} \,, \qquad m_\nu = 0.1 \text{ eV} \,, \qquad \mu = 6.0 \times 10^{-5} \text{ eV}^{-2} \,, \tag{4.20}$$

which leads to

$$\tilde{m} = 8.3 \times 10^{-4} \text{ eV} \,, \qquad m_N = 1.8 \times 10^8 \text{ GeV} \,. \tag{4.21}$$

The situation, however, is more complicated in scenarios with more than one lepton flavor. For instance, mixing could give large contributions to $\tilde{m}_1$, thereby enhancing the washout. On the other hand, it can also lead to additional sources of CP violation, which might improve the prospects for successful leptogenesis in realistic models with several flavors. Consider, for example, the situation when the third right-handed neutrino is much heavier than the other two and the mixing with the third flavor in the right-handed sector is suppressed. A novel aspect of this effective two-flavor case is that large mixing and resonant amplification of the lepton asymmetries due to the level crossing of right-handed neutrinos can enhance leptogenesis. These effects are similar to those discussed above in the full three-flavor framework. We will study the regime with a large hierarchy between the two lightest right-handed neutrinos, which allows a simple analytic approach. As a toy example, we consider the following scenario: We assume maximal mixing in the light neutrino sector and one complex phase in P_l, which can be moved into the Yukawa coupling matrix y by rephasing the electron neutrino field. Thus, the neutrino mass matrix is taken to have the form

$$m_\nu = \begin{pmatrix} e^{2i\kappa}\,\bar{m} & e^{i\kappa}\,\delta m \\ e^{i\kappa}\,\delta m & \bar{m} \end{pmatrix} \tag{4.22}$$

with $\delta m \ll \bar{m}$. The parameters $\bar{m}$ and δm can be determined from the mass of the lightest active neutrino m_0 and Δm_{21}^2:

$$\bar{m} \simeq m_0, \quad \delta m \simeq \frac{\Delta m_{21}^2}{4 m_0}\,. \tag{4.23}$$

Numerical analysis indicates that the most interesting region in the parameter space corresponds to the situation when the smaller eigenvalue of g is in the large μ regime, while the larger eigenvalue is in the small μ regime, i.e.

$$\frac{4\,y_1^2}{\bar{m}^2} \ll \mu \ll \frac{4\,y_2^2}{\bar{m}^2}\,, \tag{4.24}$$

and we will assume this to hold in the present example. In this case, two solutions for g are, to first order in λ, given by the ansatz[3]

$$g = U^\dagger \begin{pmatrix} \bar{m} & 0 \\ 0 & \pm\frac{y_2}{\sqrt{\mu}} + \frac{\bar{m}}{2} \end{pmatrix} U^*, \quad U = \begin{pmatrix} e^{-i\kappa} & \lambda e^{-i(\phi+\kappa)} \\ -\lambda e^{i\phi} & 1 \end{pmatrix} \tag{4.25}$$

with

$$\lambda = \mp\frac{\delta m\sqrt{\mu}}{y_2}\,, \tag{4.26}$$

$$\sin(\phi+\kappa) \simeq \mp\sin(2\kappa)\frac{y_1}{\bar{m}\sqrt{\mu}}\,, \tag{4.27}$$

and thus, we find

$$\tilde{m}_1 = \frac{y_1^2 + y_2^2\lambda^2}{2\bar{m}\mu} = \frac{y_1^2 + \delta m^2\mu}{2\bar{m}\mu}\,, \tag{4.28}$$

$$\epsilon_{N_1} = \frac{3}{32\pi\tilde{m}_1}\left[\sin(2\phi+2\kappa)\,\bar{m}\,\delta m^2\mu + \sin(4\kappa)\bar{m}y_1^2\right]. \tag{4.29}$$

The second term in ϵ_{N_1} essentially coincides with the corresponding expression in the one-flavor case. Hence, in this case, it is possible to generate a sufficient lepton asymmetry in

[3] The other two solutions do not lead to successful leptogenesis.

exactly the same way as in the one-flavor case as long as the contribution from mixing to $\tilde{m}_1$ does not lead to a strong washout. The latter condition reads

$$\frac{\delta m^2}{2\bar{m}} \simeq \frac{(\Delta m_{21}^2)^2}{32 m_0^3} \lesssim 10^{-3}\ \mathrm{eV}\,, \tag{4.30}$$

which is easily satisfied if $m_0 > 10^{-3}\,\mathrm{eV}$. It is interesting to note that for $\kappa = \pi/8$ and quasi-degenerate neutrino masses, the obtained asymmetry ϵ_{N_1} saturates the upper limit obtained in ref. [30].

But even in the case $\kappa \simeq \pi/4$, when the second term in the expression for ϵ_{N_1} in eq. (4.29) is suppressed, the first term can lead to viable leptogenesis. The corresponding contribution to η_B takes its largest value when $\delta m^2 = y_1^2/\mu$, so that eqs. (4.28) and (4.29) become

$$\tilde{m}_1 = \frac{y_1^2}{\bar{m}\mu}, \tag{4.31}$$

$$\epsilon_{N_1} = \frac{3}{16\pi} y_1 \bar{m}\sqrt{\mu}. \tag{4.32}$$

In this case, η_B is smaller than it is in the one-flavor case only by a factor

$$\frac{y_1}{2\bar{m}\sqrt{\mu}} = \frac{1}{2}\sqrt{\frac{\tilde{m}_1}{\bar{m}}} \simeq 0.1. \tag{4.33}$$

It should be noted that the baryon asymmetry increases with the parameter μ, so that, depending on the Yukawa couplings, saturation of the upper limit on μ in eq. (4.24) might be necessary, which can lead to deviations from our analytic results.

Thus, in the two-flavor case, two different sources of leptogenesis exist: The first source is similar to that in the one-flavor case, which is related to the type II seesaw term and is sensitive to the high-energy CP-violating phases, while the second source results from mixing effects and has no analogue in the one-generation case.

In the three-flavor framework, sources of both types are, in general, present as well, but mixing with the third flavor can further increase $\tilde{m}_1$. Figure 11 shows the baryon-to-photon ratio η_B when an additional phase is attributed to the electron neutrino, as in the two-flavor example of eq. (4.22). We choose the phase $\kappa = \pi/4$ ($\kappa = \pi/8$), so that the source similar to the first (second) term in eq. (4.29) gives the largest contribution to the baryon asymmetry. Our numerical results indicate that, similarly to the two-flavor case, the upper bound on the decay asymmetry found in ref. [30] can be saturated. The mass of the lightest right-handed neutrino that is required to reproduce the observed baryon asymmetry is $m_{N_1} \gtrsim 1.4 \times 10^9$ GeV ($m_{N_1} \gtrsim 2.5 \times 10^8$ GeV). These bounds can be relaxed by choosing Yukawa couplings different from those of the up-type quarks. With an appropriate choice, the results for the four solutions of the type '$\pm\pm+$' agree with the analytic predictions of the two-flavor analysis presented in this section. Notice that the results in the two-flavor case in eqs. (4.28) and (4.29) do not depend on y_2 as long as the constraint (4.24) is fulfilled. Likewise, we observe in the numerical analysis of the three-flavor case that in this limit leptogenesis is not very sensitive to the two largest eigenvalues y_2 and y_3. This

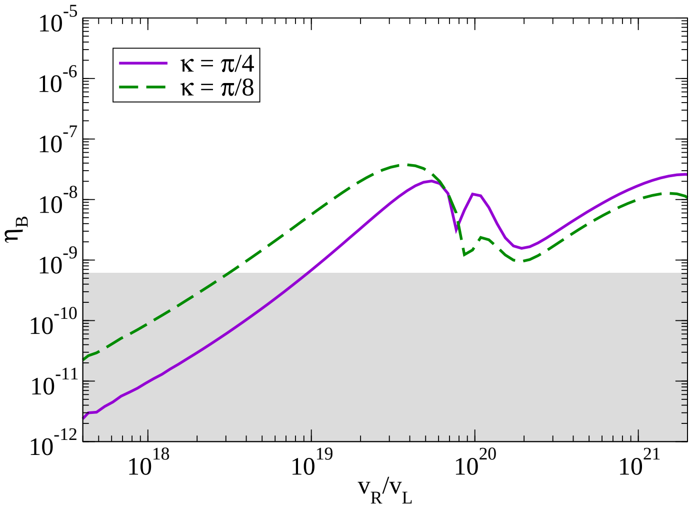

Figure 11: The baryon-to-photon ratio η_B with an additional complex phase $\pi/8$ or $\pi/4$ attributed to the electron neutrino for the solution '$--+$'. The shaded area corresponds to values of η_B below the observed value. Inverted mass hierarchy, $m_0 = 0.1\,\text{eV}$.

	$\pm++$	$\pm-+$	$\pm\pm-$
Stability	$v_R/v_L > 10^{18}$	$v_R/v_L \simeq 10^{20}$	disfavored
Leptogenesis	$v_R/v_L > 10^{18}$	$v_R/v_L > 10^{18}$	excluded
Gravitinos	$v_R/v_L < 10^{21}$	unconstrained	unconstrained

Table 1: The allowed regions of the parameter v_R/v_L for the eight different types of solutions.

is, however, a consequence of the fact that the mixing in the 1-3 sector of the Dirac-type Yukawa coupling y is small in our framework according to eq. (2.6). If this mixing is sizable, $\theta^q_{13} \gtrsim 5°$, and depending on the other parameters determining the Yukawa coupling y and the neutrino mixing matrix U_{PMNS}, leptogenesis might be suppressed, mainly due to a large contribution to the effective mass parameter $\tilde{m}_1$ from the eigenvalue y_3 and the resulting increased washout.

Thus, we conclude that successful leptogenesis is possible for four out of the eight solutions provided that the value of the electron-type Majorana phase is in an appropriate range. For the other four solutions, leptogenesis is not viable, as was first pointed out in ref. [15]. The reason for this is that, as long as the Dirac Yukawa coupling matrix is chosen to coincide with that of the up-type quarks, the mass of the lightest right-handed neutrino never exceeds 10^6 GeV and no level crossings occur. We note that in the left-right symmetric case with type I+II seesaw mechanism the bounds on the mass of the lightest right-handed neutrino can be slightly relaxed compared to those in the pure type I case which, for right-handed neutrinos with thermal initial abundance and hierarchical masses, requires $m_{N_1} \gtrsim 5 \times 10^8$ GeV [35–37].

5. Summary and conclusions

We have analyzed the left-right symmetric type I+II seesaw mechanism with a hierarchical

Dirac mass term motivated by GUTs. It was previously shown that a reconstruction of the mass matrix of heavy right-handed neutrinos in this framework produces eight solutions which result in exactly the same low-energy phenomenology. Our goal was to discriminate among these solutions using their stability properties and leptogenesis as additional criteria. As a measure of the stability, we have chosen the parameter Q which quantifies the degree of fine-tuning necessary to obtain a given mass matrix of light neutrinos and was defined in eq. (3.1). For three lepton generations, no fine-tuning corresponds to $Q \sim 10$. We have selected the value $Q = 10^3$, which corresponds to a fine-tuning at the percent level, as a maximal allowed value. The leptogenesis criterion we used was the ability of a given solution to reproduce the observed baryon asymmetry of the Universe.

Our results complement the results of the leptogenesis analysis performed in ref. [15] in the following aspects. In the case without additional Majorana phases, we obtain, in accordance with ref. [15], that a sizable decay asymmetry ϵ_{N_1} is possible close to the mass degeneracy of the two lightest right-handed neutrinos. However, while for the specific parameters used in ref. [15] the washout is too large to allow viable leptogenesis, we find that assuming the inverted mass hierarchy for the light neutrinos resolves the problem, as shown in figure 9. Similarly, in the cases with additional CP-violating Majorana phases we found that for certain solutions the choice of the parameters made in ref. [15] leads either to a strong washout (solutions '$\pm - +$'), or to a violation of the gravitino bound (solutions '$\pm + +$'). In section 4, we presented a systematic study showing that those problems can be solved for the four solutions '$\pm \pm +$' if the value of the of electron-type Majorana phase is in the appropriate range. In particular, the upper bound on the decay asymmetry for the type I+II seesaw model found in ref. [30] can be saturated for a certain choice of the parameters. This is illustrated by the analytic results for the two-flavor case in eqs. (4.28) and (4.29) and the numerical results for the three-flavor case in figure 11. We would like to emphasize that if the Dirac-type Yukawa coupling matrix y is characterized by hierarchical eigenvalues and rather small mixing, successful leptogenesis is quite a generic feature of the left-right symmetric seesaw models.

Our findings are summarized in table 1. One can observe that the stability criterion disfavors the four solutions of the type '$\pm \pm -$' and restricts the solutions of the type '$\pm - +$' to the region of the parameter space where $v_R/v_L \simeq 10^{20}$. The remaining two solutions of the type '$\pm + +$' are stable, provided that $v_R/v_L \gtrsim 10^{18}$. We found that successful leptogenesis is possible for the four solution of the type '$\pm \pm +$' as long as $v_R/v_L \gtrsim 10^{18}$. This possibility requires the existence of additional Majorana-type phases which are absent in the pure type I seesaw framework. Further constraints come from the potentially dangerous overproduction of gravitinos in supersymmetric theories, giving rise to an upper bound on the lightest right-handed neutrino mass. For our choice of the Yukawa couplings, $y = y_u$, only the solutions of the type '$\pm + +$' are affected by this constraint, which leads to the requirement $v_R/v_L \lesssim 10^{21}$. For the other six solutions, the smallest right-handed neutrino mass is always below 10^{10} GeV, so that these solutions are not constrained by this criterion. In the cases when the middle eigenvalue of y is chosen to be significantly larger than the one in our framework, $y_2 \gtrsim 10^{-2}$, the constraint $v_R/v_L \lesssim 10^{21}$ would also apply to the two solutions of the form '$\pm - +$'. On the other hand, a very small middle eigenvalue, $y_2 \lesssim 5 \times 10^{-4}$, would render leptogenesis impossible for these two solutions, since the decay asymmetry would be too small due to the small mass of the lightest right-handed neutrino.

Thus, we have shown, within the chosen framework, that the stability and leptogenesis criteria partially lift the eight-fold degeneracy among the solutions for the mass matrix of heavy right-handed neutrinos in the left-right symmetric type I+II seesaw.

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References

[1] P. Minkowski, *$\mu \to e\gamma$ at a rate of one out of 1-billion muon decays?*, *Phys. Lett.* **B 67** (1977) 421.

[2] M. Gell-Mann, P. Ramond, and R. Slansky, *Complex spinors and unified theories*, in *Supergravity* (P. van Nieuwenhuizen and D. Freedman, eds.), p. 315, North Holland, Amsterdam (1979), Print-80-0576 (CERN).

[3] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories*, O. Sawada and A. Sugamoto, eds. Tsukuba, Japan, p. 95 (1979), KEK report 79-18.

[4] S.L. Glashow, *The future of elementary particle physics*, *NATO Adv Study Inst. Ser. B Phys.* **59** (1979) 687.

[5] R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[6] M. Magg and C. Wetterich, *Neutrino mass problem and gauge hierarchy*, *Phys. Lett.* **B 94** (1980) 61.

[7] G. Lazarides, Q. Shafi and C. Wetterich, *Proton lifetime and fermion masses in an SO(10) model*, *Nucl. Phys.* **B 181** (1981) 287.

[8] J. Schechter and J.W.F. Valle, *Neutrino masses in* $\mathrm{SU}(2) \times U(1)$ *theories*, *Phys. Rev.* **D 22** (1980) 2227.

[9] R.N. Mohapatra and G. Senjanović, *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev.* **D 23** (1981) 165.

[10] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett.* **B 174** (1986) 45.

[11] W. Buchmüller, P. Di Bari and M. Plümacher, *Some aspects of thermal leptogenesis*, *New J. Phys.* **6** (2004) 105 [hep-ph/0406014].

[12] W. Buchmüller, R.D. Peccei and T. Yanagida, *Leptogenesis as the origin of matter*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 311 [hep-ph/0502169].

[13] A. Strumia, *Baryogenesis via leptogenesis*, `hep-ph/0608347`.

[14] E.K. Akhmedov and M. Frigerio, *Seesaw duality*, *Phys. Rev. Lett.* **96** (2006) 061802 [`hep-ph/0509299`].

[15] P. Hosteins, S. Lavignac and C.A. Savoy, *Quark-lepton unification and eight-fold ambiguity in the left-right symmetric seesaw mechanism*, *Nucl. Phys.* **B 755** (2006) 137 [`hep-ph/0606078`].

[16] M. Maltoni, T. Schwetz, M.A. Tórtola and J.W.F. Valle, *Status of global fits to neutrino oscillations*, *New J. Phys.* **6** (2004) 122 [`hep-ph/0405172`].

[17] A. Strumia and F. Vissani, *Implications of neutrino data circa 2005*, *Nucl. Phys.* **B 726** (2005) 294 [`hep-ph/0503246`].

[18] G.L. Fogli, E. Lisi, A. Marrone, A. Palazzo, and A.M. Rotunno, *Global analysis of neutrino masses and mixing*, *Prog. Part. Nucl. Phys.* **57** (2006) 71.

[19] J.C. Pati and A. Salam, *Unified lepton-hadron symmetry and a gauge theory of the basic interactions*, *Phys. Rev.* **D 8** (1973) 1240.

[20] H. Georgi, *The state of the art – gauge theories (talk)*, *AIP Conf. Proc.* **23** (1975) 575.

[21] H. Fritzsch and P. Minkowski, *Unified interactions of leptons and hadrons*, *Ann. Phys. (NY)* **93** (1975) 193.

[22] K.S. Babu and R.N. Mohapatra, *Predictive neutrino spectrum in minimal SO(10) grand unification*, *Phys. Rev. Lett.* **70** (1993) 2845 [`hep-ph/9209215`].

[23] G. Anderson, S. Raby, S. Dimopoulos, L.J. Hall and G.D. Starkman, *A systematic SO(10) operator analysis for fermion masses*, *Phys. Rev.* **D 49** (1994) 3660 [`hep-ph/9308333`].

[24] Particle Data Group collaboration, W.M. Yao et al., *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.

[25] E.K. Akhmedov and M. Frigerio, *Interplay of type-I and type-II seesaw contributions to neutrino mass*, *JHEP* **01** (2007) 043 [`hep-ph/0609046`].

[26] A.Y. Smirnov, *Seesaw enhancement of lepton mixing*, *Phys. Rev.* **D 48** (1993) 3264 [`hep-ph/9304205`].

[27] M. Tanimoto, *Seesaw enhancement of neutrino mixing due to the right-handed phases*, *Phys. Lett.* **B 345** (1995) 477 [`hep-ph/9503318`].

[28] G. Altarelli, F. Feruglio and I. Masina, *Large neutrino mixing from small quark and lepton mixings*, *Phys. Lett.* **B 472** (2000) 382 [`hep-ph/9907532`].

[29] T. Hambye and G. Senjanović, *Consequences of triplet seesaw for leptogenesis*, *Phys. Lett.* **B 582** (2004) 73 [`hep-ph/0307237`].

[30] S. Antusch and S.F. King, *Type II leptogenesis and the neutrino mass scale*, *Phys. Lett.* **B 597** (2004) 199 [`hep-ph/0405093`].

[31] WMAP collaboration, D.N. Spergel et al., *First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: determination of cosmological parameters*, *Astrophys. J. Suppl.* **148** (2003) 175 [`astro-ph/0302209`].

[32] A. Pilaftsis and T.E.J. Underwood, *Electroweak-scale resonant leptogenesis*, *Phys. Rev.* **D 72** (2005) 113001 [`hep-ph/0506107`].

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[33] M. Kawasaki, K. Kohri and T. Moroi, *Big-bang nucleosynthesis and hadronic decay of long-lived massive particles*, *Phys. Rev.* **D 71** (2005) 083502 [astro-ph/0408426].

[34] P.-H. Gu, H. Zhang and S. Zhou, *A minimal type-II seesaw model*, *Phys. Rev.* **D 74** (2006) 076002 [hep-ph/0606302].

[35] S. Davidson and A. Ibarra, *A lower bound on the right-handed neutrino mass from leptogenesis*, *Phys. Lett.* **B 535** (2002) 25 [hep-ph/0202239].

[36] W. Buchmüller, P. Di Bari and M. Plümacher, *Cosmic microwave background, matter-antimatter asymmetry and neutrino masses*, *Nucl. Phys.* **B 643** (2002) 367 [hep-ph/0205349].

[37] G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, *Towards a complete theory of thermal leptogenesis in the SM and MSSM*, *Nucl. Phys.* **B 685** (2004) 89 [hep-ph/0310123].